Computing Black-Hole Collisions in Einstein's Theory with Feynman Integrals

High precision black hole scattering from worldline quantum field theory: The second self-force order

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In Short

- Precise analytical predictions for the collision of black-holes or neutron stars are required for data analysis in future gravitational wave detectors.
- The problem is studied in a perturbative expansion in Newton's constant G employing quantum tools from elementary particle physics.
- This approach breaks up the scattering process into smaller manageable pieces represented by Feynman diagrams leading to Feynman integrals.
- The integration of millions of high-dimensional (four-loop) Feynman integrals of planar and non-planar topology is required.
- For this the integration-by-parts software KIRA will be employed.

When two black holes collide, they create ripples in the fabric of space-time known as gravitational waves. These were predicted by A. Einstein as early as 1918, and experimentally detected almost a century later in 2015, opening a new chapter in astronomy. Today, scientists routinely observe these cosmic collisions using specialized observatories like LIGO, Virgo, and KAGRA, with over 100 events detected so far.

We will establish more precise theoretical predictions for these collisions in preparation for the next generation of gravitational wave detectors coming online in the 2030s. These advanced instruments will measure gravitational waves with unprecedented precision - up to 100 times more accurately than current detectors. To make the most of these improved measurements, we need to develop correspondingly precise theoretical predictions.

We aim to perform extremely detailed calculations of black hole collisions using Einstein's theory of general relativity. The challenge lies in the complexity of Einstein's field equations - these are highly nonlinear partial differential equations describing both the motion of the scattered black-holes and the emitted gravitational waves. They are generically impossible to solve exactly. While computer simulations can provide accurate results, they require enormous computing power. A single simulation can take 10-50 thousand processor hours, making it impractical to generate the millions of waveform profiles needed for analyzing gravitational wave data.

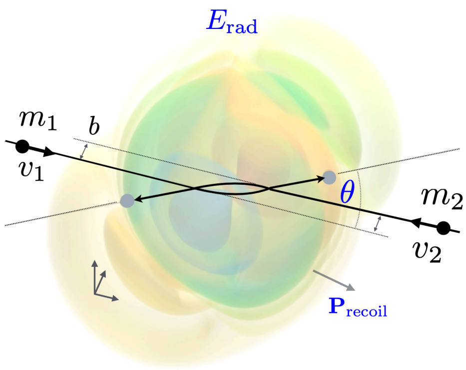

***Figure 1:** Black-hole scattering event with masses $m_{1,2}$ and incoming velocities $v_{1,2}$, impact parameter b and resulting relative scattering angle θ, radiated gravitational wave energy and the recoil momentum.*

Instead, we employ an alternative entirely analytical approach by using mathematical techniques borrowed from quantum physics: As long as the black-holes are well separated (prior to the encounter) one may expand the equations in the strength of the gravitational field controlled by Newton's constant of gravity G. In doing so we break down the problem into smaller, manageable pieces. This allows us to adapt advanced computational methods from quantum field theory, which were initially developed to describe and predict scattering processes of elementary particles (such as they occur at proton collisions in the Large Hadron Collider at CERN). In particular, we apply the method of **Feynman diagrams**, which allows us to easily visually represent and organize the various terms that contribute to our calculations, as seen in Fig. 2. We use these now to predict the scattering of black-holes, neutron stars or stars in our universe.

Using this approach, significant progress has already been made, going well beyond the traditional methods of general relativity applied to black hole collisions. In fact, our team recently achieved unprecedented fifth-order calculations in the G expansion [1,2], providing extremely accurate predictions for certain scenarios.

Our current proposal aims to complete the final piece of this mathematical puzzle. Concretely, our

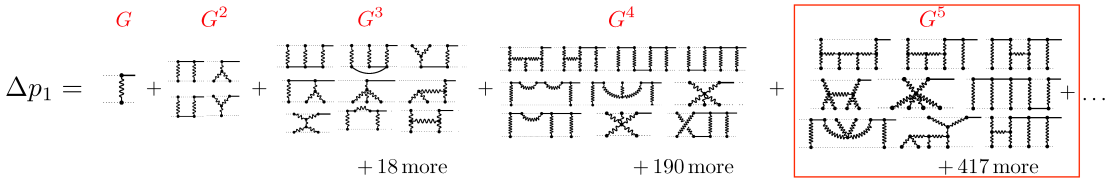

Figure 2: *Feynman diagrammatic expansion of a two black-hole scattering process (here change of momentum). The wiggly lines represent the gravitational interactions, the top and bottom lines the black-hole trajectories and deflections. Every single diagram translates to specific high-dimensional integrals.*

fifth order results are only valid if one of the black holes is much larger than the other, as it is the case for example in the scattering of a massive object off a supermassive black hole in the center of a galaxy. Yet the case where the two black holes are of a similar size, which are precisely the type we typically observe in our gravitational wave detectors, needs to be computed. This is the goal of our project. This would provide the most precise theoretical predictions yet for black hole collisions, matching the needs of next-generation gravitational wave detectors. Our work has already yielded surprising connections to complex mathematical structures called Calabi-Yau manifolds, which also occur in string theory and are a research topic in pure mathematics (algebraic geometry).

However, the calculations involved are monumentally complex. Within each Feynman diagram, there are tens of thousands of so-called Feynman integrals, each of which is an integral over sixteen variables. These are a challenge to solve efficiently, particularly if there are millions of integrals involved as is the case here. We will use our advanced understanding of these types of integrals, specialized software, and the significant computational resources at NHR@ZIB to tackle this problem.

The impact of our research will be far-reaching. This work represents a crucial step forward in our ability to understand and predict gravitational wave signals from black hole collisions, preparing us for the next generation of gravitational wave astronomy in the 2030s and beyond. Our results will help scientists **better interpret gravitational wave signals from future detectors**, which will in turn provide valuable data for cosmologists answering the big questions about the state of our universe. We will **improve our understanding of black hole physics**, which are by far the most mysterious and interesting objects out there in the universe. We will potentially reveal **new connections between gravity and quantum physics**. And finally, we will **advance mathematical techniques** for solving complex physics problems relevant to particle physics.

Through our calculations, we will bridge the gap between theoretical physics and observational astronomy, enabling more precise measurements and deeper insights into the nature of gravity and the universe itself.

WWW

http://qft.physik.hu-berlin.de

More Information

[1] M. Driesse, G. U. Jakobsen, G. Mogull, J. Plefka, B. Sauer and J. Usovitsch, “Conservative Black Hole Scattering at Fifth Post-Minkowskian and First Self-Force Order,” Phys. Rev. Lett. **132** (2024) no.24, 241402 [arXiv:2403.07781 [hep-th]]. doi:10.1103/PhysRevLett.132.241402

[2] M. Driesse, G. U. Jakobsen, A. Klemm, G. Mogull, C. Nega, J. Plefka, B. Sauer and J. Usovitsch, “High-precision black hole scattering with Calabi-Yau manifolds,” [arXiv:2411.11846 [hep-th]]; under review in Nature. doi:10.48550/arXiv.2411.11846

Project Partners

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DFG Subject Area

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