

Measuring turbulence from within: a quasi-Lagrangian approach

Quasi-Lagrangian studies of spatio-temporal correlation in incompressible MHD turbulence

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In Short

- We run numerical studies of three-dimensional incompressible turbulence in neutral fluids and electrically conducting fluids (plasmas).
- Turbulent flows are composed of energetic structures of various sizes, which (in 3D turbulence) break up to ever smaller ones before being dissipated to heat at the smallest scales
- We compute the correlation between velocity and magnetic field fluctuations on different length scales and with a time lag between them to deduce temporal and spatial properties of these nonlinear transfer processes
- We adopt the quasi-Lagrangian frame of reference to measure the fluctuations: tracer particles carry a set of probes at fixed distances, at which the velocity relative to the particle position is measured.

Turbulent flows are fundamental to various physical processes throughout the universe. On Earth, they can be observed in oceans, the atmosphere, smoke from a chimney or in the wake of an airplane. Examples in astrophysics include stellar winds, the generation of magnetic fields in celestial bodies, the propagation of cosmic rays and star formation from molecular clouds.

Turbulent flows can be viewed as being composed of vortices of various sizes which interact with each other in a complex, so-called “nonlinear” way, leading to chaotic, unpredictable behaviour. In three-dimensional turbulence, large vortices tend to break up into successively smaller structures, thereby transferring energy from large to small scales. At the smallest scales, the energy is converted into heat by viscous dissipation.

The underlying differential equations are the incompressible Navier-Stokes equations for neutral fluids and the magnetohydrodynamic (MHD) equations, which describe a plasma as a single electrically conducting fluid. Due to the complexity of the MHD equations, a comprehensive theory of MHD turbulence has not been formulated up to the present day. We aim at expanding our understanding of turbulence with direct numerical simulations (DNS), in which the governing equations are solved with a pseudo-spectral method.

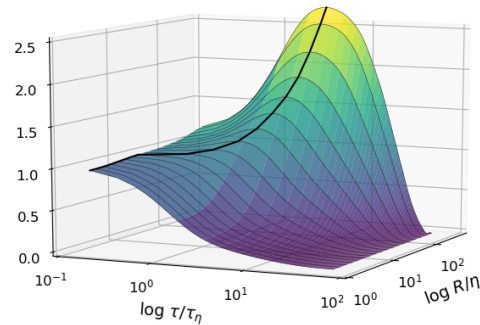


Figure 1: Spatio-temporal correlation of fluctuations of varying length scale R and with small-scale fluctuations at a delayed time, determined by the time lag τ . The black line marks the time lag at which the correlation is maximal for a given R .

In this project we compute spatio-temporal correlation functions of velocity and magnetic field fluctuations, to gain further insight into the properties of the nonlinear transfer processes. This was done before in the hydrodynamic case [1] and we have verified those results at a higher resolution and used this simpler case to establish a framework with which to analysis the correlation between fluctuations of different length scales. In a next step, we have extended this approach to MHD turbulence. The presence of a large-scale magnetic field makes this system anisotropic, i.e. the plasma behaves differently in the directions parallel and perpendicular to the magnetic field. Therefore, we computed the fluctuations in both of these directions and have varied the strength of the mean magnetic field to study its influence as well.

During a simulation, we record a time series of the velocity and magnetic field fluctuations, from which the correlation between two fluctuations on certain length scales can be computed. These measurements typically are taken either in the standard Eulerian picture (fixed frame of reference) or the Lagrangian picture (co-moving with the fluid), as visualized in figure 2. While the former is suited for the computation of spatial correlations, the temporal aspect is not directly accessible due to the advection of small-scale structures by large-scale motions. In the Lagrangian frame of reference, trajectories of tracer particles are considered, which are suitable for the computation of temporal fluctuations but deliver only a subset of spatial information bound to the tracer trajectories. The so-called quasi-Lagrangian frame [2] combines both aspects in a controllable and well-defined approximation, by employing tracer particles which carry along with them a set of probes

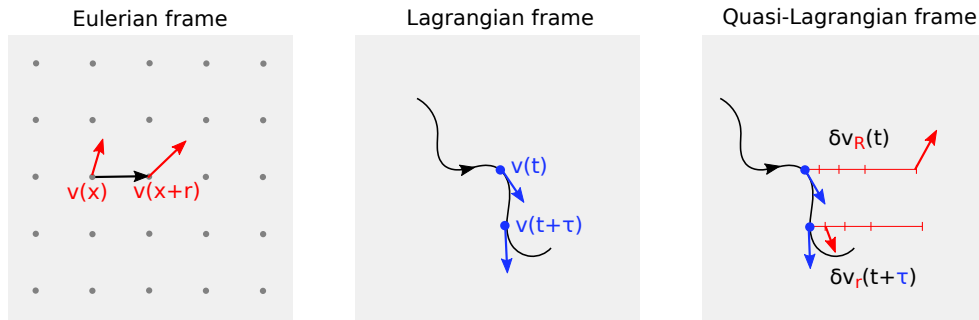


Figure 2: Schematic figure of the Eulerian, Lagrangian and quasi-Lagrangian frames of reference.

at fixed distances from the particles. The difference in velocity at the particle's position and the probe positions gives rise to the velocity fluctuations. The magnetic fluctuations are obtained similarly.

By introducing a time lag between the time series of those two fluctuations, we can measure the temporal correlation. Due to the chaotic nature of turbulence, we expect the correlation to become zero as the time lag becomes comparable to the so-called "large-eddy turnover time". This time scale estimates how long it takes for a large eddy to transfer its energy down to the dissipative scales and represents the longest time scale in the turbulent flow. Similarly, by varying the length scales of the fluctuations, we have access to the spatial correlation. Together, this information allows us to deduce transfer processes occurring on certain time and length scales. An example of this for the hydrodynamic case is shown in figure 1, in which the length scale of the velocity fluctuation at the delayed time is set to the smallest scale of the system. For a certain range of length scales, the correlation peaks at some finite time lag. This indicates that the large-scale fluctuations at a preceding time are correlated with the small-scale fluctuation at a delayed time, a manifestation of the direct energy cascade.

In the MHD case particularly, we can use our results on the temporal aspects of the energy transfer to compare to predictions made by different phenomenological theories. For example, we have measured the maximum time lag in our correlation function for various scale pairs (R, r) and plotted it against the difference in scales $|R - r|$, to see how long on average it takes for energy to be transferred from structures of scale R to those of scale r . The result for the direction perpendicular to the magnetic field is shown in figure 3 for different mean magnetic field strengths B_0 . A power law scaling can be observed in a certain range, which can be compared to predictions from literature, e.g. to [3].

In short: by measuring the turbulence from within the flow, we aim at gaining further insights into the

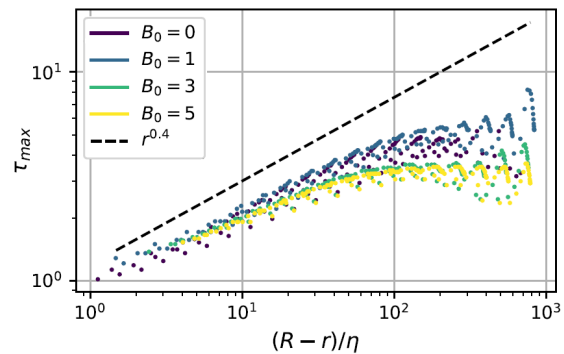


Figure 3: Time lag at maximum correlation for all scale pairs (R, r) in MHD turbulence with different mean magnetic field strength B_0 .

properties of the nonlinear transfer processes to help building a comprehensive theory of MHD turbulence.

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More Information

- [1] L. Biferale, E. Calzavarini, F. Toschi, *Physics of Fluids* **23.8**, 085107 (2011). doi: 10.1063/1.3623466
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